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Portfolio Optimization Using Ant Colony Method a Case Study on Tehran Stock Exchange

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Portfolio Optimization and selection of the efficient frontier from Mean-Variance Markowitz(1952) model is easily accessible in the conditions with no constraints.However, it cannot be used to fulfil the investors' needs based on having different constrains such as number of the assets in a portfolio. Since the Markowitz model is not the answer to these investors, there should be other methods to provide the optimal risk and return combination. Therefore, Meta-Heuristic methods have become a highly active area of research in this field. This paper tries to construct portfolios rooted in constrains regarding the number of assets involved in a portfolio having the inspiration of Ant Colony Algorithm; moreover, the study is targeted to find the best efficient frontier for the proposed algorithm. Next, sharp ratio has been employed as the fitness function of the portfolio which tries to optimize portfolio selection by optimizing the sharp ratio. In addition, an efficient frontier for ant colony algorithm has been set and demonstrated that the algorithm functions more efficiently when we confine the number of assets to a precise number.

JEL Codes: E62, P33, E27

1. Introduction

Portfolio selection problem has continuously been one of the most important topics of research in modern finance. Assembling assets to maximize expected return over the minimized risk is one of all investors' distresses. The problem is mostly concerned with allocating capital over a few available assets. The term referring to investors as risk-averse means that in case of two portfolios having the same level of return, the investor selects the one having the lowest level of risk. The main goal of the portfolio selection is to select the best combination of assets that yields the highest expected returns, while at the same time, ensuring an acceptable level of risk(Mokhtar et al. 2014). Considering investors' needs, one ought to maximize return while minimizing the risk of a portfolio. However, high returns are generally comprised of increased risk. Diversification is another term that is more favorable among investors. Diversified portfolios are more favorable among investors because they can reduce their exposure to one asset risk by holding combination of assets with no correlation with one another.

Modern portfolio theory as a mathematical framework became accessible to produce the best combination of mean and variance. The classic mean-variance model was first introduced by Markowitz in 1952 which was considered as the foundation of the modern portfolio theory. The basic model obtains efficient frontier, as the best portfolio of assets that achieves a predetermined level of expected return at the minimal risk.

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For every level of the desired mean return or risk, this efficient frontier indicates the best investment strategy. The results of investigations were dissatisfying for modifying Markowitz model to meet the investors' needs. The basic Markowitz model includes no cardinality constraint around number of assets and investors' needs on a portfolio. In other words, the investors having specific assets in their portfolio and specific amount of capital cannot use the Mean-Variance model to obtain their best diversification and combination of portfolios. Such constraints in portfolio optimization problems have created the need to search for more accurate solution with respect to Linear Programing (LP) problems and constraints diversity.

Optimization is one of the best solutions. The optimization is a mathematical procedure that helps a process to be more functional. Heuristic and Meta-heuristic models are two techniques for solving problems that classical models are incapable to solve them. One of the solutions for Portfolio Optimization Problems (POP) is Meta-heuristic approach.

This paper has used Ant Colony Optimization theory (ACO) to optimize the sharp ratio of portfolios having constrains on number of assets. In addition, as discussed in this paper, the current inspection procedure leads us to graphs in which efficient frontier of portfolios constructed by Markowitz model and the ACO models can be compared. The remaining parts of this paper are organized as follows. Section 3 introduces the problem struggling with the diversification of portfolio, and defines ant colony optimization and the implantation of algorithm for portfolio optimization. Section 4 provides the computational tests of the algorithm in which Markowitz model has been used as a benchmark and the results are presented as well. Section 5 derives the conclusion.

2. Literature Review

Meta-Heuristics algorithm is one of the best algorithms that have been discussed recently; however, there have been numerous studies on the usage of these algorithms in the field of portfolio optimization. Bacanin et al. (2014) have used Bee Colony in order to solve the portfolio optimization under constrains of number of assets involved in a portfolio. They have compared the algorithm with genetic and firefly algorithms. The results announce the efficiency of the algorithm for portfolio optimization. Kuoand Hong(2013) presented a two-stage method of investment portfolio based on soft computing techniques. The first stage uses data envelopment analysis to select most profitable funds, while hybrid of genetic algorithm (GA) and particle swarm optimization (PSO) is proposed to conduct asset allocation in the second stage. The evaluation of results shows that Sharpe's value of portfolio based on the proposed method is superior to those of portfolio based on the GA, PSO and market index. The proposed method really can robustly assist investors to obtain gains. Nigam and Agarwal (2013)have presented a comparison between ant colony algorithm and genetic algoritm for index fund; moreover in the impirical study, the ant colony has performed more satisfactory than the genetic algorithm.Deng and Lin(2010) used the ant colony algorithm in the USA stock exchange,london stock exchange as well as in Japan,Germany and Hong Kong stock exchange to solve the problems under cardinaly constraints. The results presented reveal that ant colony algorithm can perform more efficientlycompared tothe PSO especially for portfolios having lower risk. Forghandoost & Kazemi(Haqiqi and Kazemi,2012)presented an approach on the ACO and used Tehran stock exchange data set to show that the algorithm is suitable for portfolio optimization;however according to the difference between the portfolio that has been constructed by the ACO and the optimum value, the method is not always reliable. Doerner et al. (2006)chose thePACO and compared it with Pareto Simulated Annealing , Non-Dominated Sorting Genetic Algorithm; furthermore, they solved the ACO by adding a pheromone vector to a specified objective function. They presented that the ACO performs more efficiently than the other algorithms.

3. Problem Description

As mentioned in the introduction of this paper, one of the problems that most of the investors are struggling with is to use the best combination of the risk and return to yield the best diversification of the portfolio. Diversification by itself is not able to solve the problem. A few investors have a specific amount of capital to invest. Some of them need their portfolios to contain specific assets. There are always a need for the optimized diversification. This optimization can come true using different methods which can find the best combination of assets to meet the investors' goals. Regarding this study, itis based on the investors' demand on the capital constrains and specific assets. The study aims to achieve an efficient frontier for a specified number of assets in a portfolio and compare it to the Markowitz model. In addition, standard deviation of errors of the difference between Markowitz model and the ACO in consistent risk are calculated having the intention to show the efficiency of the methodology.

3.1 Markowitz Mean-Variance Model

Initially, mean-variance analysis generated relatively little interest; however, after a short time period, the financial community adopted the thesis. Today, financial models are constantly being reinvented to incorporate new findings based on those very same principles (Fabozzi et al., 2007). The most important role of the Markowitz theory is to set up the best combination of risk and return for investors' decisions. Markowitz constructed a mathematical approach by defining the risk as a quantitative criterion.

3.2 Portfolio Risk and Return

In a multi-objective optimization problem, multiple-objective functions need to be optimized simultaneously (Chaharsooghi and Kermani,2008).Let N be the number of different asset, r_i is the expected return of asset i $(i = 1,...,\mathrm{N})$, σ_{ij} stands for the covariance between assets i and j $(j = 1,...,N)$, the decision variable x_i represents the proportion $(0 \le x_i \le 1)$ of the portfolio invested in asset i using this notation; we can present that(Elton et al., 2009):

$$\max R_p = \sum_{i=1}^{N} r_i x_i \tag{1.1}$$

$$\min \sigma_p = \sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \sigma_{ij}} \tag{1.2}$$

$$\sum_{i=1}^{N} x_i = 1 \qquad (1.3)$$

Equation (1.3) ensures that the whole available capital is invested.

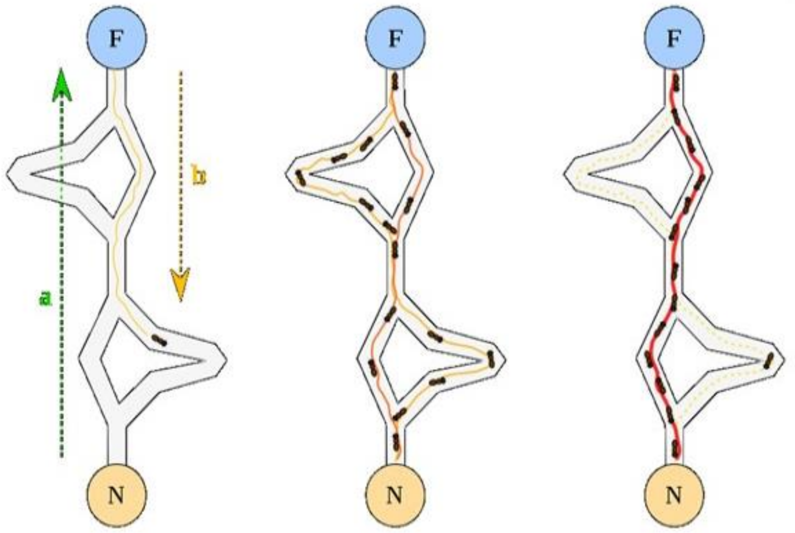
3.3 Optimization

Mathematical Optimization is a technique targeted to choose the best possible component of a collection. Whether there is a constraint or not, we can select different methods. While finding the optimal solution is very complicated, heuristic models can be used to find a solution. The solution found by the heuristic models may not be perfect and completely optimal but it is sufficient and leads us to our goal. Moreover, heuristic models lead us to the solution at a faster pace. In case of having large sets of data, we may use Meta-heuristic models. These models may not globally lead us to the optimal solutions; however, the solutionsare sufficient. The methodology of these algorithms is such a way that they find some samples; next, they go through those samples to find the local optimal solution.

3.4 Ant Colony Algorithm

Ant colony optimization is a technique for optimization which was introduced in the early 1990’s. The inspiring source of ant colony optimization is the foraging behavior of real ant colonies(Blum,2005). This algorithm designed for the first time by Dorigo et al. (1996)by the inspiration of ants path through their nest and food. One of the natural events is related to ants moving behavior to find food; in such algorithm, factors are artificial ants or they are ants which are acting like the real ants.Ant colony algorithm is a lustrous example of cumulative intelligence in which the factors that are not sufficient individually act sufficiently if they work as a team. In this algorithm,the objective of every ant is to find the shortest path between two nodes of a graph in which the problem is defined(Dorigo, 2006). Ants always look for the shortest path between their nest and food. When they move through an edge, they pour some pheromone on that edge. This amount of pheromone is supposed to be constant and shown by $\Delta\tau$ in the simplified algorithm(Dorigo, 2006); therefore by this phenomenon, ants deposit increase the probability of the path that they trace on it.As a result, in the future, other ants have more desire to choose this path to reach the food. During the evaporation, the pheromones evaporate with a constant coefficient; this leads the pheromone on the shortest path retaining much more than the longer path and it is chosen by the larger number of the ants. The pheromone evaporation abides the algorithm to find an inapt solution. The solution that is found in every round also improves the communal information of the team despite of quality of the solution. The experiments shows that if the graph becomes more elaborate, producing more than two paths between the nest and the food, algorithm behavior does not have the constancy as before;thus it is responsive to the parameters values(Dorigo et al.1996). The figure 1 shows the selection procedure by a group of ants:

Figure 1: Gross Experiments on Argentines Ants



The main idea of this algorithm is to show the self-governing behavior that is able to generalize the selection for optimal portfolio and can be used for selecting the assets involved in portfolios. Different aspects of ants' behaviorreveal different algorithms. Ant colony algorithms have been recognized as one of the most successful ant algorithms inspired by the ants foraging behavior.

3.5 Implementation of Algorithm for Portfolios

Harry Markowitz Mean-Variance model is known as the base of the portfolio optimization for determining the efficient frontier; albeit this model does not acquit the investors' needs as number of assets in portfolio constraint. In this paper,the ant colony optimization algorithm aims to fulfill the need to optimize the portfolio of stocks. The proposed algorithm is a recurring strategy; in the first step of the algorithm, ants enter in a graph with nodes that are claimed to be the number of assets in investors' portfolio. The edge which is the connector between these nodes represents the utility of this combination. Edges retain pheromone information achieved by previous repeats which are recalled in matrix$[\tau_{ij}]_{N \times N}$. τ_{ij} representing the amount of pheromone poured between 2 nodes i and j . The amount and density of the pheromone on an edge is a factor for investigating the utility of an edge and the probability to be selected with other ants in order to create shorter paths. The pheromone information, saved in every edge of the graph, is used randomly for the next path selection.

At the beginning of the algorithm, every edge has the same amount of pheromone equal to τ_0 . If between 2 nodes of i and j , the node i represents the asset which has been selected by the ant number K ,and the node j represents the asset that has not been selected till now;the probability of choosing the jth asset by the ant number K is calculated by(Maringer, 2006):

(1.4)

$$p_{ij}^{k} = \begin{cases} \dfrac{\tau_{ij}}{\sum\limits_{m \in N_i} \tau_{im}} & , j \in N_i \\ 0 & , j \in N_i \end{cases}$$

In which the probability to select the asset that have been selected before is equal to zero(Maringer, 2006).

The next step is to determine the appropriate coefficient for building an optimized portfolioin order to optimize asset selection. The optimized coefficients are achievable by calculating $\begin{bmatrix} a & b \\ b & c \end{bmatrix}$(Roll,1977) :

(1.5)

$$A = \begin{bmatrix} a & b \\ b & c \end{bmatrix} = \begin{bmatrix} R'\sigma_{ij}^{-1}R & R'\sigma_{ij}^{-1}I \\ R'\sigma_{ij}^{-1}I & I'\sigma_{ij}^{-1}I \end{bmatrix}$$

R Represents the return of assets
σ_{ij} Represents the covariance between assets i and j
And I represents a matrix with $N \times 1$ dimension with elements equal to 1
At last the coefficient vector X_T is calculated as:

(1.6)

$$x_T = \sigma_{ij}^{-1} RIA^{-1} \begin{pmatrix} r_p \\ I \end{pmatrix}$$

To evaluate every portfolio, the sharp ratio is calculated as:

(1.7)

$$SR_p = \frac{R_p - r_s}{\sigma_p}$$

At the end, the amount of pheromone on an edge ought to be updated. In this step, there are two processes of evaporation and updating the pheromone. The pheromone which are deposited on every edge of the graph is calculated as follows (Maringer, 2006):

(1.8)

$$\Delta\tau_{ij} = \frac{Q_\mu}{L_{ij}}$$

Q_μ represents the amount of pheromone and L_{ij} is equal to the paths lengths; this constrains shows that the amount of pheromone deposited on a path is fitted with the quality of the answers that an ant has found.

In this approach, the algorithm aims to maximize the sharp ratio;thus instead of path lengths, we have:

(1.9)

$$L_{ij} = \frac{1}{SR_p}$$

Therefore, for the $\Delta\tau_{ij}$:

(1.10)

$$\Delta\tau_{ij} = Q_\mu.SR_{ij}$$

To amplify the algorithm power for finding the optimized portfolio, a rating system have been proposed that assigns the highest rank to the portfolio having the highest sharp ratio(Bullnheimer et al.):

(1.11)

$$Q_\mu = ((\omega - \mu) + 1).Q$$

In the above formula, ω represents the number of ants and μ represents rank of the sharp ratio compared to other portfolios. This equation ensures that the ant having the highest sharp ratio has the most pheromone depositing on the path;thus this accelerates the process of finding the optimal solution.

The evaporation is always assumed as a descending function. The conventional method for evaporation is the exponential function; in which in every round, a positive number less than one is multiplied to the pheromone amount(Maringer, 2006):

(1.12)

$$\tau = (1-\rho)\tau \quad \rho \in [0 \quad , \quad 1]$$

4. Empirical Study

4.1 Research Data

In this paper, the data have been extracted from Teheran Stock Exchange. For this approach,the top 50 indexeshave been chosen and the data are the price of these 50 companies between 09/25/2011 and 09/22/2014. Prices and the index values have been converted to monthly returns. Fourteen companies were removed according to their availability in this time period and 36 of the companies were used in calculations.

For those 36 data, risk and return and covariance between data were calculated. Risk free rate were chosen to be equal to short term bank deposits, 17% (1.316 % monthly).

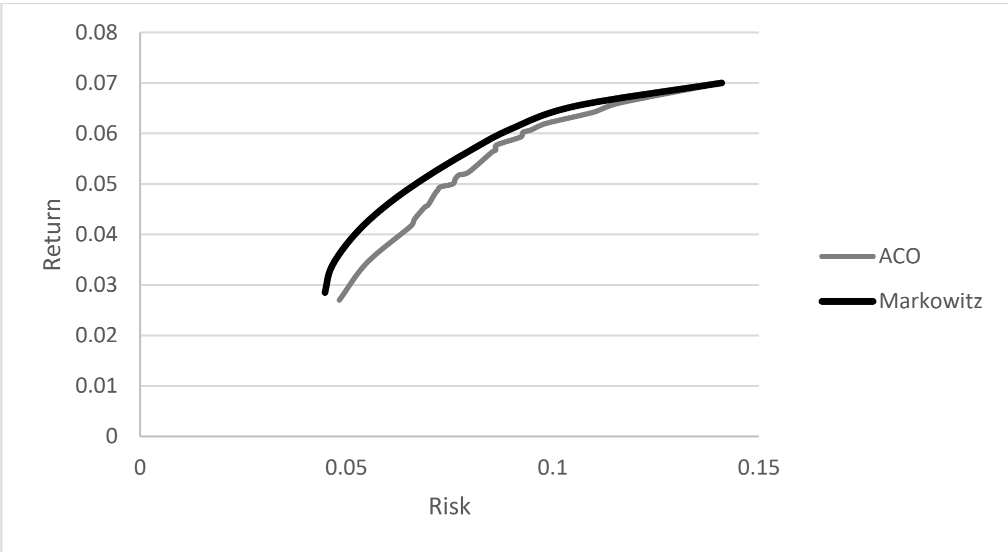
4.2 Computational Tests

To assess the performance of the ACO algorithm, the Markowitz model was used as a benchmark. We compared different combinations of risk and return obtained from the ACO with the results of Markowitz model. To achieve this, we solved the Markowitz model for data gathered from Tehran stock exchange and the efficient frontier was sketched. This method was performed by Portopt function and the different combination of risk and return in Matlab are sketched(Seidlov_É and Po__ivil). To obtain the ACO efficient frontier, the algorithm was run in several steps using different number of portfolios and through different portfolios; hence, the portfolio with the highest return in the same risk level was chosen. The parameters of ACO algorithm were obtained as follows:

-number of ants equal to 100,
- $Q = 0.1$,
- τ_0 =0.1,
- $\rho = 0.5$.

To obtain the value of the parameters, the algorithm was run independently for several times and the parameters which led to the best results were chosen. Subsequently, efficient frontier of the Markowitz model was sketched in comparison of the ACO algorithm as shown in the figures below:

Figure 2: Efficient Frontier for 5 ACO and Markowitz



Several tests were run on the constraint of number of assets and the result was compared with the Markowitz efficient frontier as presented in tables below; moreover,to compare the consistent value of variance, tests were run on the ACO model and the highest return on every variance value were selected.For that specific value of variance, an interpolation search was accomplished on Markowitz model and these two returns were sketched inone figure:

Results for 5 Assets:

Figure 3: Efficient Frontier for 5 Assets

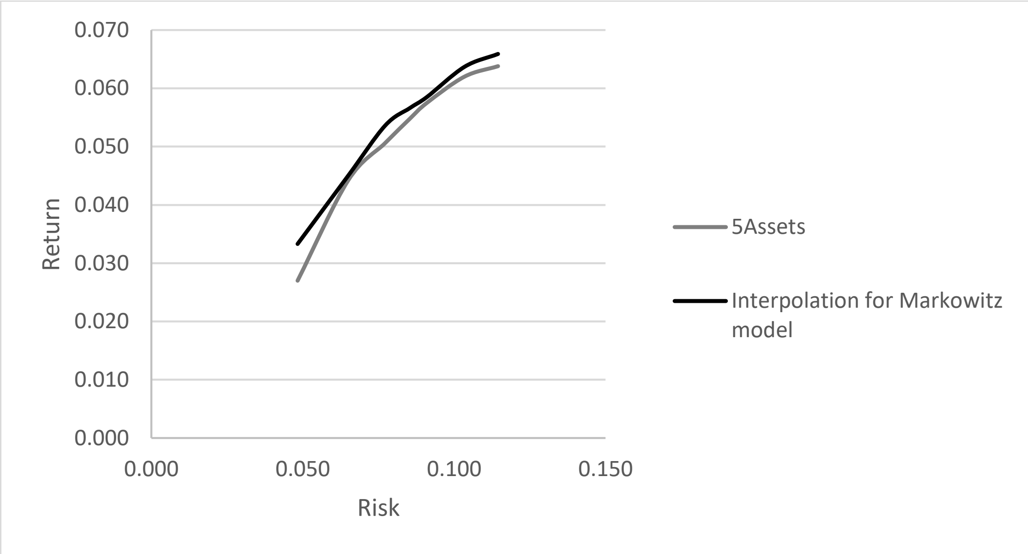


Table 1.Risk and Return Comparison 5 Assets

comparison between Markowitz results and ACO for 5 assets							
σ	0.048	0.065	0.077	0.085	0.091	0.103	0.115
R ACO	0.027	0.044	0.051	0.055	0.058	0.062	0.064
R Markowitz	0.033	0.045	0.054	0.057	0.059	0.064	0.066

Result for 10 Assets:

Figure 4: Efficient Frontier for 10 Assets

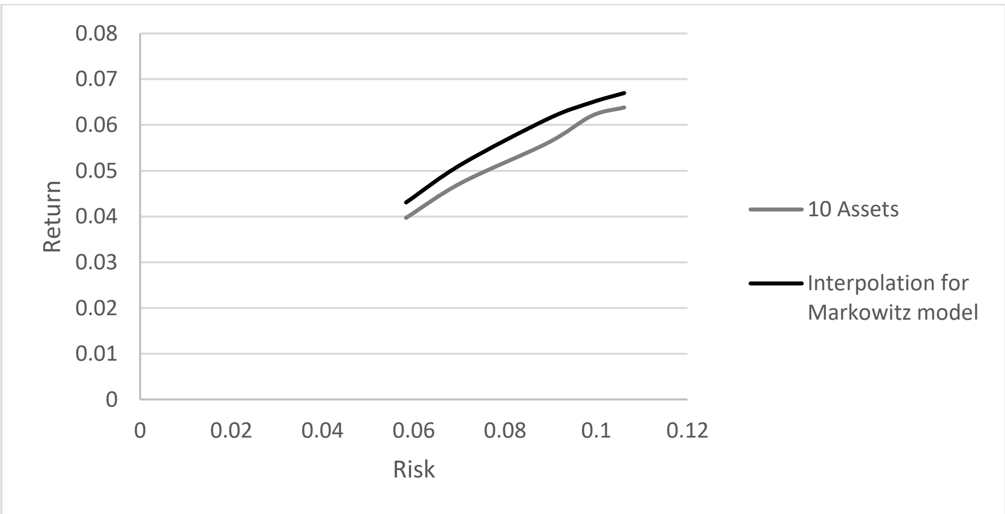


Table 2: Risk and Return Comparison 10 Assets

comparison between Markowitz results and ACO for 10 assets					
σ	0.0583	0.0705	0.0894	0.099	0.106113
R ACO	0.0397	0.0474	0.0561	0.062	0.063827
R Markowitz	0.043058	0.051437	0.061341	0.064939	0.066982

Result for 15 Assets

Figure 5: Efficient Frontier for 15 Assets

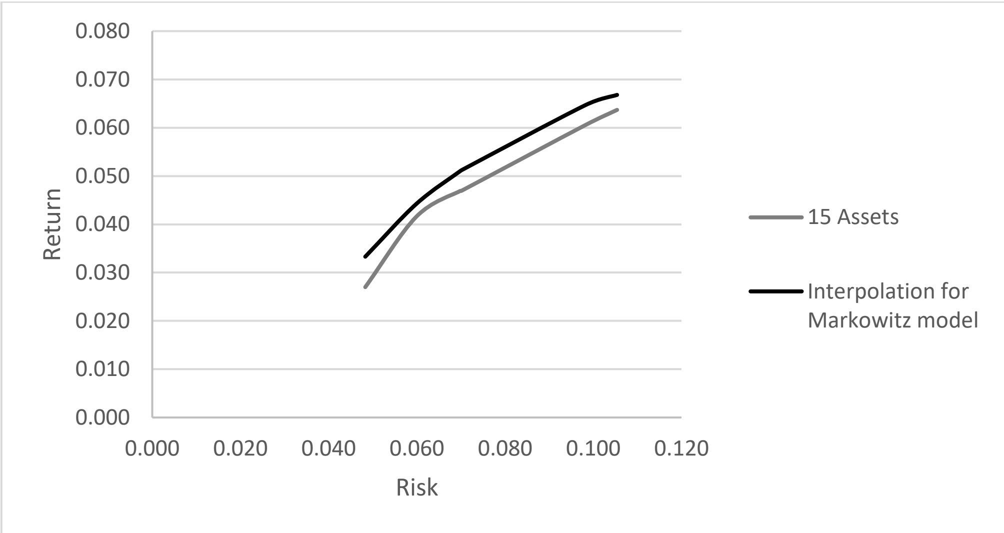


Table 3: Risk And Return Comparison 15 Assets

comparison between Markowitz results and ACO for 15 assets						
σ	0.048	0.060	0.070	0.070	0.098	0.105
R ACO	0.027	0.042	0.047	0.047	0.061	0.064
R Markowitz	0.033	0.044	0.051	0.051	0.065	0.067

Result for 20 Assets:

Figure 6: Efficient Frontier for 20 Assets

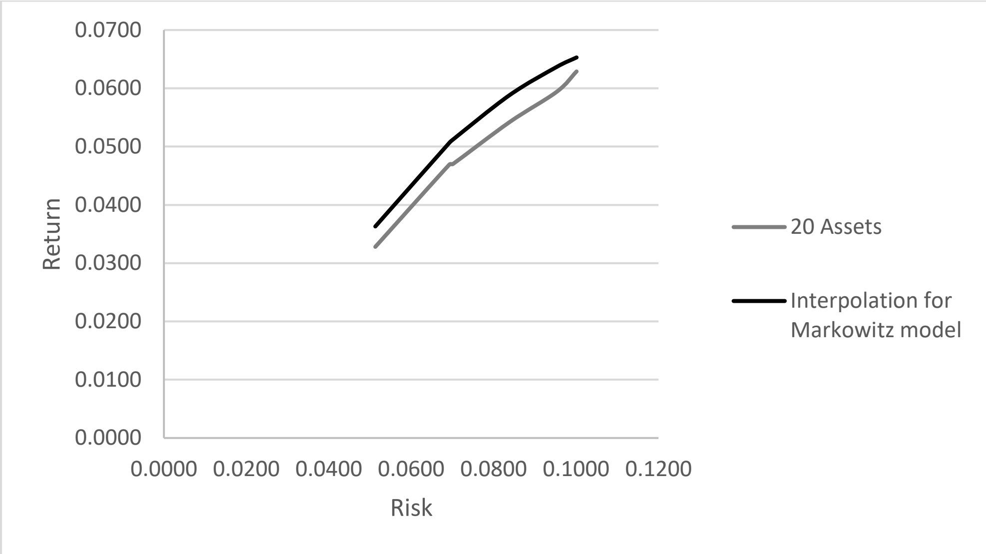


Table 4: Risk and Return Comparison 20 Assets

comparison between Markowitz results and ACO for 20 assets						
σ	0.0513	0.0693	0.0701	0.0839	0.0953	0.1001
R ACO	0.0328	0.0469	0.0470	0.0542	0.0594	0.0629
R Markowitz	0.0363	0.0507	0.0512	0.0588	0.0637	0.0653

Result for 25 Assets:

Figure 7: Efficient Frontier for 25 Assets

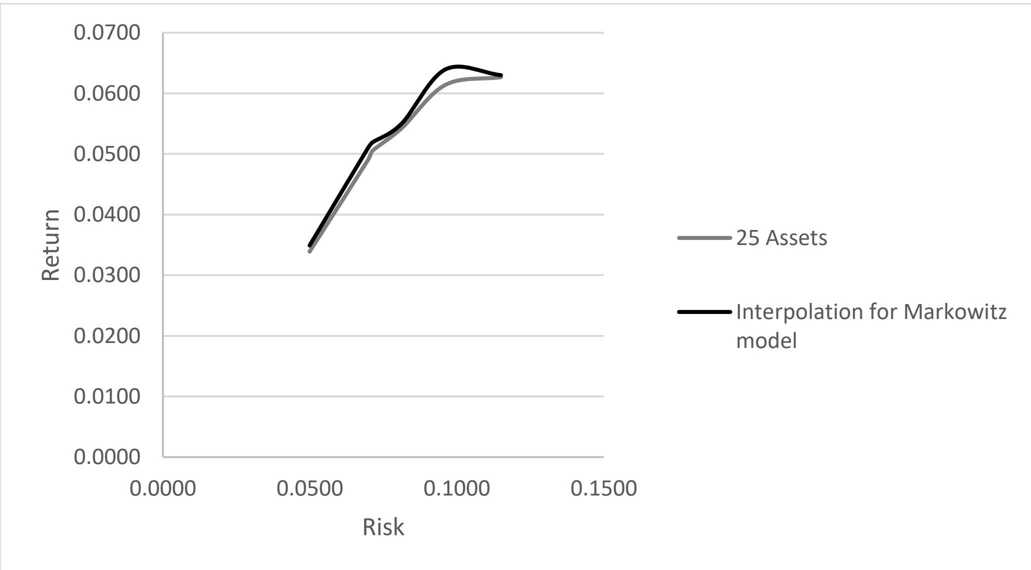


Table 5: Risk and Return Comparison 25 Assets

comparison between Markowitz results and ACO for 25 assets						
σ	0.0499	0.0697	0.0712	0.0812	0.0961	0.1150
R ACO	0.0339	0.0490	0.0505	0.0542	0.0614	0.0627
R Markowitz	0.0349	0.0509	0.0519	0.0550	0.0640	0.0630

Table 6: Standard Deviation of the Errors

number of assets	5	10	15	20	25	30
standard deviation of the errors	0.001872	0.000931	0.001263	0.000785	0.000819	0.000419

As depicted in figure 3 to 7 the Markowitz efficient frontier and the ACO line for the numbered assets are tracing the same degree of risk and return which shows the reliability of the ACO model. Also as table 1 to 5 shows the calculated σ, R ACO and R Markowitz, with every σ the difference between R ACO and R Markowitz is subtle which again illustrates the reliability of the model. As illustrated in these figures, diversification of the portfolio is reduced till specific number of assets is obtained; afterwards, adding asset to a portfolio does not impact the risk.Table 6 points the standard deviation of the errors which also illustrates the high coherence between R ACO and R Markowitz.

5. Conclusion

In this paper, an ant colony optimization method was demonstrated for constrained portfolio selection problems.The ant colony is the well fitted solution for problems that attempts to generate new components and adds them to the state as well as for problems having dynamic combinatorial optimizations. In this optimization method, convergence is guaranteed. The model points out a sharp ratio that is ensured in every point of riskassisting the investorsto have a portfolio with the best return. The case study was observed on the data set extricated from Tehran stock exchange. The results delineated that the ACO algorithm finds a portfolio significantly close to Markowitz efficient frontier along with investors' needs on the number of assets involved in the portfolio. The findings of this study are restricted to the sharp ratio, number of assets and capital constrains for each asset in order to evaluate the best portfolio; moreover, it is obvious that altered ratio ends in the altered results. The proposed algorithm is flexible and extendable in other constraints such as the maximum investments on each asset.As future studies, researches can be expanded to apply the ACO and other constraints that were used in portfolio optimization.

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