Portfolio Selection Using Fuzzy Analytic Hierarchy Process (FAHP)

Zahra Lashgari\(^1\) and Kobra Safari\(^2\)

Investors participate in stock markets for the exclusive purposes of earning highest return and sustaining and enhancing profitability. With this in mind, identifying effective determining factors regarding the selection of both the volume and type of invested securities (portfolio) is a matter of great importance. The selection of an optimal portfolio is dependent upon different factors, including extrinsic and intrinsic criteria that are associated with investors’ objectives. The present study first attempts to identify the most effective factors utilized in the selection of a portfolio; then, taking advantage of the Fuzzy Analytical Hierarchy Process (FAHP) as one of the multi-criteria decision-making (MCDM) models, the study proceeds to demonstrate the selection of an optimal stock portfolio. In this paper, in order to investigate the criteria that are most effective in the selection of stocks, a hierarchy of criteria is first presented. Using the Delphi methodology which recognizes the value of experts’ opinions, the best available advice from experts is selected and analyzed, and the weight of each identified criterion is measured applying the Analytic Hierarchy Process (AHP) method. The findings of this research suggest that the most significant criteria in the decisions made by investors regarding the selection of stocks are international risk, international economy, competitiveness (as extrinsic factors), and profit, management quality, information transparency, efficiency (as the criterion associated with investors’ objectives and as intrinsic factors). Finally we made a selection (portfolio) consisting of 7 top companies with the highest total return between 2007- 2011. Then, regarding the result of previous method which were based on both quantitative and qualitative criteria, we ranked the stock of those companies using Fuzzy Analytic Hierarchy Process (FAHP).

JEL Codes: E22 and G11

1. Introduction

One of the most important features of industrial and developed countries is the availability of an active and dynamic money and capital market. In other words, when the savings of the people are directed into the production sector using proper mechanisms while bringing some returns to the owners, they can also be useful for the implementation of economic plans of the society as the most significant factor in financing. If they are entered into unsound courses, it will result in some adverse effects in the society; in the countries where the volume of liquidity in the hands of people is high, this issue is seen more than anywhere else.

The important task of economic officials of the country is to direct and attract such liquidity and to create ideal conditions for producing higher returns on such money resources for the entire society. The stock exchange is one of the most important instruments capable of attracting such liquidity, for the owners purchasing the stocks can operate their capital with a return as expected and participate in financing across the country as well.

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he portfolio selection issue has been a matter of concern to the specialists of investment affairs and the investors for a long time. In other words, all the investors are trying to achieve the best possible choices by meeting appropriate criteria in making investment decisions and considering their personal preferences.

Although the identification of the decision-making process of the investors is difficult, it seems necessary to investigate it. This is because by clarifying which criteria is considered and what information is required by the investors when they are selecting a specified stock, they can be helped in making rational decisions for investment. This includes rendering assistance to the government in privatization and enforcing economic policies as well as offering more to corporations under their ownership.

Another significant point in equity investment is that decision-making is not merely carried out based on an ideal criterion, but it is done using several criteria commonly and simultaneously. These criteria might be quantitative or qualitative; for making decisions in such cases, it is better to use multi-criteria decision-making models that can incorporate quantitative and qualitative criteria. Now the question is: what these criteria (inter-organizational, intra-organizational, and personal objectives of investors) could be, and how the importance of each of the identified criteria (quantitative and qualitative) is valued by the investors?

The analytical hierarchy process (AHP) method presented by Tomas L. Saati in 1970 is one of the best known and most practical multi-criteria decision-making models. This model is used when it is required to consider quantitative and qualitative aspects of the decision simultaneously (Rao and Davim, 2008; Verma and Koul, 2008). The AHP technique will reduce the complexity of decisions applying paired comparisons (one-to-one) and combining the outcomes. Accordingly, it helps the decision-makers to arrive at the best decision and presents a rational and clear justification that is indicative of being the best decision (Rao and Davim, 2008). Within the AHP context, the decision maker cannot provide deterministic preferences and uses perception-based judgment intervals instead. This kind of uncertainty in preferences can be modeled using fuzzy set theory (Verma and Koul, 2008). The fuzzy set theory approach could resemble human reasoning in the use of approximate information and uncertainty to generate decisions (Hsu, 2010). In the fuzzy set terminology, the ratio supplied by the decision maker is a fuzzy number described by a membership function. Here, a membership function describes the degree with which elements in the judgment interval belong to the preference set. Fuzzy AHP consists of deriving the local priorities from these fuzzy preference ratios, which are subsequently aggregated to form the global priorities. The fuzzy AHP computes fuzzy priorities based on arithmetic operations for fuzzy triangular (or trapezoidal) numbers (Verma and Koul, 2008). This method has the ability to capture the vagueness of human thinking style and effectively solve multi-criteria decision making problems (Naghadehi et al., 2009). Another significant question that arises here is that how is it possible to obtain the best equity portfolio from the given firms using the FAHP method while considering the weight assigned to the identified quantitative and qualitative criteria in previous research?

Conducted in Tehran Stock Exchange (TSE) on equity portfolio selection, only the quantitative criteria were taken into account and the decision-making criteria were not classified. Therefore, there seems to be a lack of a study that uses fuzzy theory for equity portfolio selection; because, in the circumstance where the main decision maker is a human being, the use of fuzzy logic would bring about more reassured and tangible results.
Considering the significance of the subject and the features of AHP and FAHP techniques, the purpose of this research study includes three main objectives:

First, identifying extrinsic and intrinsic criteria and the investors’ objectives in portfolio selection; then, specifying the degree of importance of each identified criteria in the previous stage using the analytical hierarchy process method; and finally selecting the optimal stock portfolio applying the fuzzy analytical hierarchy process method.

In order to provide an answer to the research questions and realize the research objectives, the factors influencing the priorities of investors regarding equity portfolio selection are addressed in the first section using Delphi and regression methods. Then, in the second section, the weight of each identified criterion is measured applying the Analytic Hierarchy Process (AHP) method. Finally, taking such criteria into consideration and using the FAHP method, we proceed to select a stock portfolio include the top seven companies listed in the Tehran Stock Exchange.

2. Literature Review

Historically, the mean-variance model is the first example of a portfolio optimization problem, and it is credited to Markowitz, who presented his ideas in 1952. This model is important because mean-variance analysis provides a basis for the derivation of the equilibrium model known variously as the Capital Asset Pricing Model (CAPM), Sharpe-Lintner model, black model and two-factor model. Furthermore, by using AHP Saaty in 1980 and Durer et al. in 1997 analyzed more complex portfolio systems. To select securities using data envelopment analysis (DEA), Teriyaki in 2001 evaluated the financial performance of companies whose securities are traded on the ISE (Teriyaki, 2009). Tanaka and Guo (1999) also proposed two portfolio selection models based on fuzzy probabilities and possibility distributions, rather than conventional probability distributions as in Markowitz’s model. Ehrgott (2004) applied a developed model of Markowitz mean variance to select the optimal portfolio. In this study, to resolve the portfolio optimization problem, non-convex programming of integers is applied. Lacagnina and Pecorella (2006) developed a multistage stochastic fuzzy program with soft constraints and recourse in order to capture both uncertainty and imprecision and used their program to solve a portfolio management problem. Using Sharpe’s single-index model in a soft framework, Huang (2006) revised the conventional mean-variance method to determine the optimal portfolio selection under conditions of uncertainty. Ballestero (2007) in a study in Frankfurt Stock Exchange introduced a method in the first phase where the stocks that are inefficient according to the previous data are eliminated. The second phase of this method makes use of a decision schedule prepared with several scenarios of uncertainty conditions the cells of this schedule measure the results by the multi-criteria linear function index of simulated outcomes of the next period. Zhag and Others (2007) proposed two kind of portfolio selection models based on lower and upper possibility means and possibility variance and presented an algorithm which can derive the possibility efficient frontier of the problem. Model Huang (2008) is a method based on hybrid artificial intelligence algorithms. The results achieved in this study show that the suggested method is very efficient in providing very strong decision-making parameters and in resolving the optimization problem. A hybrid artificial intelligence method is used when the optimization problem cannot be resolved by analysis. Teriyaki (2009) has used the FAHP method for portfolio selection. In this study, the model is based on the two models presented by Enea and Pieza and the population of the study is the Istanbul Stock Exchange (ISE) According to the results of this study, both of the
methods applied delivered the same data to the investors for ranking and weighing through the FAHP method.

Since most of the studies on portfolio optimization in our country have focused on quantitative criteria only; in this study we aim through the use of experts’ opinion in Delphi Technique to identify both quantitative and qualitative criteria that are effective in portfolio optimization in TSE.

Further in next step we attempt to employ Fuzzy Analytic Hierarchy Process (FAHP) to calculate the weights of these criteria for stock ranking and portfolio selection purposes.

3. The Methodology

We proceed to identify both quantitative and qualitative criteria in three categories: internal factors, external factors and factors which are related to investors’ objectives and this leads to develop the hypothesis as below:

Primary hypothesis is that given the identified qualitative and quantitative criteria and using FAHP methods, ranking of companies in stock portfolio selection will be different from the ranking in portfolio selection based on the quantitative factors (i.e. mean-variance).

First sub-hypothesis: Inter-organizational criteria (including political, economic and technological factors) have a positive and direct impact on equity portfolio selection.

Second sub-hypothesis: Intra-organizational criteria (including company’s profit, size, and technology) have a positive and direct impact on equity portfolio selection.

Third sub-hypothesis: Investors’ objectives (profit making, security, excitement and control) have a positive and direct impact on equity portfolio selection.

In this investigation, preliminary decision-making criteria were identified in the first phase using survey methods and searching in valid databases, scientific texts related to the portfolio selection issue, portfolio selection strategies and admission criteria.

The identified criteria were classified into three groups including extrinsic criteria, intrinsic criteria and the investors’ objectives and were distributed among the experts and specialists using Delphi method. The subjects were asked to comment on the confirmation and/or rejection of the items. They were also requested to add any index to the questionnaire they proposed other than those identified.

After filling out the questionnaires, those criteria in each group (extrinsic, intrinsic and the investors’ objectives) that were accepted by the experts more than 70% were chosen as selected criteria. Then, through examining the third sub-hypotheses the final criteria were selected.

Then these criteria were classified into a hierarchy of factors related to each of these three groups. Using the AHP method, a group of criteria bearing the maximum weights were selected. For comparing the companies and selecting the optimal stock portfolio, a third questionnaire was used applying the FAHP method.
Meanwhile, since the questionnaires designed in this investigation were based on two standard methods including Delphi and AHP (modulated and standard measurement tool), they demonstrated validity and reliability. The population under study consists of all specialists, investors and experts in TSE among whom the mentioned four questionnaires were distributed and filled out by them. The research sample was randomly selected from the above population as well.

Previous research conducted in TSE on portfolio selection only considered quantitative criteria, by ignoring classification of decision-making factors. Therefore, regarding their limitations, the need for a research that introduces fuzzy theory to stock portfolio selection is felt. That is, in cases where the main decision maker is human, using fuzzy logic will lead to more reliable and concrete results.

3.1 AHP Method

The analytic hierarchy process, introduced by Saaty, addresses how to determine the relative importance of a set of activities in a multi-criteria decision problem. The process makes it possible to incorporate judgments on intangible qualitative criteria along with tangible quantitative criteria (Badri, 2001).

The AHP procedure organizes an unstructured decision problem at different hierarchical levels with the set of decision elements positioned at the bottom level, the set of decision criteria and sub criteria located at the intermediate levels, and the goal residing at the top of the decision hierarchy. Usually the decision elements represent a finite set of options or alternatives and the decision criteria represent a set of properties that are common for all the decision alternatives. The components in each level are then compared in a pair-wise fashion according to their importance using a static nine-point scale (Cakir, 2008), referred to as the “fundamental scale” by Saaty (1980). The pair-wise judgments are arranged in a positive reciprocal matrix $A = [a_{ij}]$, called the pair-wise comparison matrix (Cakir, 2008).

<table>
<thead>
<tr>
<th>Definition</th>
<th>Intensity of importance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extremely more important</td>
<td>9</td>
</tr>
<tr>
<td>Very strongly more important</td>
<td>7</td>
</tr>
<tr>
<td>Strongly more important</td>
<td>5</td>
</tr>
<tr>
<td>Moderately more important</td>
<td>3</td>
</tr>
<tr>
<td>Equally important</td>
<td>1</td>
</tr>
</tbody>
</table>

**Table 1: A Nine-point importance scale**

Source: Albayrak and Erensai (2004)

In order to control the results of the method, the consistency ratio for each of the matrices and overall inconsistency for the hierarchy are calculated.

The deviation from consistency are expressed by the following equation consistency index and the measure of inconsistency is called the consistency index (CI). The consistency ratio (CR) is computed by dividing the CI by a value obtained from a table of random consistency index (RI).
The consistency index (CI) and consistency ratio (CR) are calculated as follows:

\[
\text{C.I} = \frac{\lambda_{\text{max}} - n}{(n-1)}
\]

\[
\text{C.R} = \frac{\text{C.I}}{\text{R.I}}
\]

Where \( \lambda_{\text{max}} \) is the largest eigenvalue of the comparison matrix, \( n \) is the number of items being compared in the matrix, and RI is a random index. If the CR \( \leq 0.10 \), the comparisons are acceptable (Albayrak and Erensal, 2004).

### 3.2 FAHP Method

In this study, the extent FAHP which was originally introduced by (Chang, 1996) is utilized. The steps of Chang’s extent analysis can be given as follows:

**Step 1:** The value of fuzzy synthetic extent with respect to the ith object is defined as

\[
S_i = \sum_{j=1}^{m} M_{jgi}^{j} \times \left[ \sum_{i=1}^{n} \sum_{j=1}^{m} M_{jgi}^{j} \right]^{-1}
\]

**Step 2:** As \( M_1 = (l_1, m_1, u_1) \) and \( M_2 = (l_2, m_2, u_2) \) are two triangular fuzzy numbers, the degree of possibility of \( M_2 = (l_2, m_2, u_2) \geq M_1 = (l_1, m_1, u_1) \) is defined as

\[
V(M_2 \geq M_1) = \sup \{ \min (\mu M_1(x), \mu M_2(y)) \} \quad \text{and can be expressed as follows:}
\]

\[
V(M_2 \geq M_1) = \text{hgt}(M_1 \cap M_2) = \mu M_2(d)
\]

\[
= \begin{cases} 
1 & \text{If } M_2 \geq M_1 \\
L_2 - u_1 & \text{If } l_1 \geq u_2 \\
\frac{(m_2 - u_2) - (m_1 - l_1)}{(m_2 - u_2) - (m_1 - l_1)} & \text{Otherwise}
\end{cases}
\]

**Step 3:** The degree possibility for a convex fuzzy number to be greater than k convex fuzzy \( M_i \) \((i = 1, 2\ldots k)\) numbers can be defined by

\[
V(M \geq M_1, M_2\ldots M_k) = V((M \geq M_1) \text{ and } (M \geq M_2) \text{ and } \ldots \text{ and } (M \geq M_k)) = \min V(M \geq M_i), i=1,2,3\ldots,k
\]
Assume that $d(A_i) = \min V(S_i \geq S_k)$ for $k = 1, 2, \ldots, n$; $k \neq i$. Then the weight vector is given by

$$w = [d(A_1), d(A_2), \ldots, d(A_n)]'$$

Where $A_i$ ($i = 1, 2, \ldots, n$) are $n$ elements.

**Step 4:** Via normalization, the normalized weight vectors are

$$w = \left[ d(A_1), d(A_2), \ldots, d(A_n) \right]'$$

Where $W$ is a non-fuzzy number.

### 4. The Findings

#### 4.1 Data Analysis

**Extrinsic Criteria:** These criteria are extrinsic factors or environmental features effective on the industry performance, but the companies have no direct effect on it. These include political, economic, and technological factors.

**Intrinsic Criteria:** Such criteria are intrinsic or operational features that may show how the companies are making decisions and how much is their successful competition capacity. These factors are profitability and quality.

**Investors’ objectives criteria:** These are the values based on what the investor defines as his measures across the business world.

By a field study, 14 extrinsic criteria, 24 intrinsic criteria, and 5 other criteria for the investors’ objectives in stock portfolio selection were identified and distributed among 35 experts and specialists of TSE. They were asked to comment on the confirmation and/or
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rejection of the criteria. They were also requested to add any criteria to the questionnaires other than the identified ones. The following diagrams show the final percent acquired from each factor using the Delphi method (Diagram 1, 2, 3).

Figure 2: The verifiability percentage of each extrinsic criterion

![Diagram showing verifiability percentage of extrinsic criteria](image)

Figure 3: The verifiability percentage of each intrinsic criterion

![Diagram showing verifiability percentage of intrinsic criteria](image)
Figure 4: The verifiability percentage of each factor associated with the investors’ objectives

Once the questionnaires were filled out, those criteria in any group (including intrinsic criteria, extrinsic criteria and investors’ objectives) accepted by the experts at a level higher than 70% were chosen as selected indexes of each group. Then, to test the hypotheses, the significance of the selected criteria was examined using the test of significance for $\beta$ in linear regression and those criteria whose significance [relation] was not verified were omitted (Table 2).
Table 2: Selected Criteria based on Linear Regression

<table>
<thead>
<tr>
<th>Criteria</th>
<th>β</th>
<th>Sig</th>
<th>R2</th>
<th>Result</th>
<th>Final Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>international economy</td>
<td>0.134</td>
<td>0.012</td>
<td>0.25</td>
<td>H₀ Rejected</td>
<td></td>
</tr>
<tr>
<td>elasticity of supply</td>
<td>0.214</td>
<td>0.000</td>
<td>0.13</td>
<td>H₀ Rejected</td>
<td></td>
</tr>
<tr>
<td>elasticity of demand</td>
<td>0.180</td>
<td>0.003</td>
<td>0.23</td>
<td>H₀ Rejected</td>
<td></td>
</tr>
<tr>
<td>technology level</td>
<td>0.254</td>
<td>0.000</td>
<td>0.12</td>
<td>H₀ Rejected</td>
<td></td>
</tr>
<tr>
<td>industry type</td>
<td>1.198</td>
<td>0.000</td>
<td>0.29</td>
<td>H₀ Rejected</td>
<td></td>
</tr>
<tr>
<td>competitiveness</td>
<td>0.423</td>
<td>0.023</td>
<td>0.17</td>
<td>H₀ Rejected</td>
<td></td>
</tr>
<tr>
<td>international risk</td>
<td>0.231</td>
<td>0.010</td>
<td>0.34</td>
<td>H₀ Rejected</td>
<td></td>
</tr>
<tr>
<td>market share</td>
<td>0.651</td>
<td>0.021</td>
<td>0.27</td>
<td>H₀ Rejected</td>
<td></td>
</tr>
<tr>
<td>systematic risk</td>
<td>0.498</td>
<td>0.005</td>
<td>0.20</td>
<td>H₀ Rejected</td>
<td></td>
</tr>
<tr>
<td>EPS growth</td>
<td>0.569</td>
<td>0.000</td>
<td>0.24</td>
<td>H₀ Rejected</td>
<td></td>
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<tr>
<td>efficiency</td>
<td>0.172</td>
<td>0.045</td>
<td>0.17</td>
<td>H₀ Rejected</td>
<td></td>
</tr>
<tr>
<td>earnings per share</td>
<td>0.194</td>
<td>0.034</td>
<td>0.23</td>
<td>H₀ Rejected</td>
<td></td>
</tr>
<tr>
<td>sales growth</td>
<td>0.781</td>
<td>0.000</td>
<td>0.41</td>
<td>H₀ Rejected</td>
<td></td>
</tr>
<tr>
<td>Price to earning</td>
<td>0.131</td>
<td>0.039</td>
<td>0.16</td>
<td>H₀ Rejected</td>
<td></td>
</tr>
<tr>
<td>intrinsic value per share</td>
<td>0.234</td>
<td>0.008</td>
<td>0.18</td>
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<td></td>
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<tr>
<td>management quality</td>
<td>1.120</td>
<td>0.000</td>
<td>0.42</td>
<td>H₀ Rejected</td>
<td></td>
</tr>
<tr>
<td>Information transparency</td>
<td>1.238</td>
<td>0.005</td>
<td>0.34</td>
<td>H₀ Rejected</td>
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<tr>
<td>revenue</td>
<td>0.049</td>
<td>0.145</td>
<td>0.09</td>
<td>H₀ Not Rejected</td>
<td></td>
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<tr>
<td>Capital increase</td>
<td>0.018</td>
<td>0.245</td>
<td>0.06</td>
<td>H₀ Not Rejected</td>
<td></td>
</tr>
<tr>
<td>Share holders</td>
<td>0.188</td>
<td>0.298</td>
<td>0.04</td>
<td>H₀ Not Rejected</td>
<td></td>
</tr>
<tr>
<td>Energy dependence</td>
<td>0.129</td>
<td>0.208</td>
<td>0.04</td>
<td>H₀ Not Rejected</td>
<td></td>
</tr>
<tr>
<td>Profit</td>
<td>1.210</td>
<td>0.000</td>
<td>0.33</td>
<td>H₀ Rejected</td>
<td></td>
</tr>
<tr>
<td>Control</td>
<td>0.231</td>
<td>0.021</td>
<td>0.23</td>
<td>H₀ Rejected</td>
<td></td>
</tr>
<tr>
<td>information</td>
<td>1.109</td>
<td>0.000</td>
<td>0.28</td>
<td>H₀ Rejected</td>
<td></td>
</tr>
</tbody>
</table>

Significant level is 0.05

**H₀**: The desired criterion has no significant direct impact on the stock portfolio selection.  
**H₁**: The desired criterion has a significant direct impact on the stock portfolio selection.

All of these factors were classified hierarchically. The second phase was to specify the significance level of the criteria identified in the previous phase using the AHP method (Table 3-5)
### Table 3: Combined weight extrinsic criteria

<table>
<thead>
<tr>
<th>Factor</th>
<th>Criteria</th>
<th>weight Factor</th>
<th>weight criteria</th>
<th>Final weight criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Economic</td>
<td>international economy</td>
<td>0.45</td>
<td>0.58</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>elasticity of supply</td>
<td>0.45</td>
<td>0.22</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>elasticity of demand</td>
<td>0.45</td>
<td>0.20</td>
<td>0.09</td>
</tr>
<tr>
<td>Technological</td>
<td>technology level</td>
<td>0.21</td>
<td>0.21</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>industry type</td>
<td>0.21</td>
<td>0.26</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>competitiveness</td>
<td>0.21</td>
<td>0.53</td>
<td>0.11</td>
</tr>
<tr>
<td>Political</td>
<td>international risk</td>
<td>0.34</td>
<td>1</td>
<td>0.34</td>
</tr>
</tbody>
</table>

### Table 4: Combined weight intrinsic criteria

<table>
<thead>
<tr>
<th>Factor</th>
<th>Criteria</th>
<th>weight Factor</th>
<th>weight criteria</th>
<th>Final weight criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profitability</td>
<td>market share</td>
<td>0.57</td>
<td>0.05</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>systematic risk</td>
<td>0.57</td>
<td>0.07</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>earnings per share</td>
<td>0.57</td>
<td>0.09</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>efficiency</td>
<td>0.57</td>
<td>0.27</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>EPS growth</td>
<td>0.57</td>
<td>0.15</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>sales growth</td>
<td>0.57</td>
<td>0.10</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>Price to earning</td>
<td>0.57</td>
<td>0.16</td>
<td>0.09</td>
</tr>
<tr>
<td>Quality</td>
<td>intrinsic value per share</td>
<td>0.57</td>
<td>0.12</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>management quality</td>
<td>0.43</td>
<td>0.64</td>
<td>0.28</td>
</tr>
<tr>
<td></td>
<td>Information transparency</td>
<td>0.43</td>
<td>0.36</td>
<td>0.16</td>
</tr>
</tbody>
</table>

### Table 5: Combined weight criteria related to investors’ objectives

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Final weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit</td>
<td>0.66</td>
</tr>
<tr>
<td>Control</td>
<td>0.20</td>
</tr>
<tr>
<td>Information</td>
<td>0.14</td>
</tr>
</tbody>
</table>
Considering the multiplicity of the criteria, among the criteria in each group, those criteria having the highest weight and their total weight equal or more than 60 percent of the total weights were selected as final criteria.

Accordingly, the final criteria were as follows:

- Profit (with average weight 0.66)
- Management quality (with average weight 0.28)
- Information transparency (with average weight 0.16)
- Efficiency (with average weight 0.15)
- International risk (with average weight 0.34)
- International economy (with average weight 0.26)
- Competitiveness (with average weight 0.11)

The above criteria were used as final criteria for stock portfolio selection of the companies. However, before making use of such criteria, it was necessary to standardize the weights of the criteria. To do this, the weight of each criterion was divided by the total weights of the criteria (Table 6).

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Standard weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit</td>
<td>0.33</td>
</tr>
<tr>
<td>International risk</td>
<td>0.17</td>
</tr>
<tr>
<td>Management quality</td>
<td>0.14</td>
</tr>
<tr>
<td>International economy</td>
<td>0.13</td>
</tr>
<tr>
<td>Efficiency</td>
<td>0.08</td>
</tr>
<tr>
<td>Information transparency</td>
<td>0.08</td>
</tr>
<tr>
<td>Competitiveness</td>
<td>0.07</td>
</tr>
<tr>
<td><strong>Sum</strong></td>
<td><strong>1</strong></td>
</tr>
</tbody>
</table>

In this analysis, since the consistency index in each phase was less than 0.1, they were demonstrated as necessarily consistent.

The third phase was to select the optimal stock portfolio applying the FAHP.

Using the FAHP method, an appropriate portfolio for investment was selected considering the final criteria identified in the previous phase among seven top companies with the highest total return between 2007-2011 as follow:

S2: Behshahr Industrial Co.
S3: Eghtesad Novin Bank
S4: Chadormalu Mining & Industrial Co.
S5: Mines & Metals Development Investment Co.
S6: Pars Daru Holding
S7: Farsit Doroud Co.
Here we address the usage of FAHP technique in ranking the companies based on the international economic criterion:

Table 7: Initial pair wise comparison matrix of the companies based on the international economic criterion after data integration

<table>
<thead>
<tr>
<th>Criteria</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5</th>
<th>S6</th>
<th>S7</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>(1,1,1)</td>
<td>(1,2,3,3)</td>
<td>(3,3,6,7,5)</td>
<td>(0,3,3,4,11,7)</td>
<td>(0,1,4,0,5,7)</td>
<td>(0,2,1,8,5)</td>
<td>(1,1,3,3,2)</td>
</tr>
<tr>
<td>S2</td>
<td>(0,3,3,0,4,3,1)</td>
<td>(1,1,1)</td>
<td>(0,3,3,1,4,4,3)</td>
<td>(0,1,4,1,7,8,5)</td>
<td>(0,2,1,1,8,3)</td>
<td>(0,1,4,1,8,5)</td>
<td>(0,1,4,1,3)</td>
</tr>
<tr>
<td>S3</td>
<td>(0,2,0,2,7,0,33)</td>
<td>(0,3,3,1,4,4,3)</td>
<td>(1,1,1)</td>
<td>(0,1,4,1,11,3)</td>
<td>(0,1,4,1,7,6,5)</td>
<td>(0,1,4,1,0,9,3)</td>
<td>(0,1,1,0,4,5,2)</td>
</tr>
<tr>
<td>S4</td>
<td>(0,1,4,0,2,4,3)</td>
<td>(0,2,4,0,7,7)</td>
<td>(0,3,3,4,11,7)</td>
<td>(1,1,1)</td>
<td>(3,3,3)</td>
<td>(0,3,3,1,2,2,3)</td>
<td>(0,1,6,0,6,7,2)</td>
</tr>
<tr>
<td>S5</td>
<td>(0,1,4,0,2,5,7)</td>
<td>(0,3,3,2,7,8,5)</td>
<td>(0,2,4,7,3,7)</td>
<td>(0,3,3,0,3,3,0,33)</td>
<td>(1,1,1)</td>
<td>(3,1,3,3,5)</td>
<td>(0,1,2,0,3,6,0,5)</td>
</tr>
<tr>
<td>S6</td>
<td>(0,2,0,5,5,5)</td>
<td>(0,2,0,5,5,7,14)</td>
<td>(0,3,3,0,9,2,7,14)</td>
<td>(0,3,3,0,8,2,3)</td>
<td>(0,2,0,7,5,0,33)</td>
<td>(1,1,1)</td>
<td>(0,5,0,7,8,3)</td>
</tr>
<tr>
<td>S7</td>
<td>(0,5,0,7,5,1)</td>
<td>(0,3,3,1,7,14)</td>
<td>(0,5,2,2,2,9)</td>
<td>(0,5,1,5,6,25)</td>
<td>(2,2,7,7,8,3)</td>
<td>(0,3,3,1,3,2)</td>
<td>(1,1,1)</td>
</tr>
</tbody>
</table>

Calculation of the vector-\(Si\):

This vector is calculated by multiplication of two vectors as follows:

To obtain the first vector, we sum up the components of fuzzy numbers in each row. The second vector is the sum of all triangle numbers in the above matrix that has been inverted. This vector is the same in the calculation of all \(Si\)s. The inverse of a triangular number will be obtained as follows:

If \(a_{ij} = (l, m, u)\) is a triangular fuzzy number, its inverse will be as follows:

\[
a_{ij}^{-1} = (1/u, 1/m, 1/l)
\]

According to the above descriptions, \(Si\) vectors will be calculated as follows:

\[
S_1 = (6.67, 18, 29, 23, 23) \times \left(\frac{1}{166.02}, \frac{1}{68.37}, \frac{1}{29.21}\right) = (0.040, 0.270, 0.79)
\]

\[
S_2 = (2.28, 8.75, 2.1) \times \left(\frac{1}{166.02}, \frac{1}{68.37}, \frac{1}{29.21}\right) = (0.010, 0.130, 0.72)
\]

\[
S_3 = (2.06, 7.13, 17, 33) \times \left(\frac{1}{166.02}, \frac{1}{68.37}, \frac{1}{29.21}\right) = (0.010, 0.100, 0.59)
\]

\[
S_4 = (5.16, 18, 29, 17, 33) \times \left(\frac{1}{166.02}, \frac{1}{68.37}, \frac{1}{29.21}\right) = (0.030, 0.270, 0.59)
\]

\[
S_5 = (5.12, 12, 98, 25, 83) \times \left(\frac{1}{166.02}, \frac{1}{68.37}, \frac{1}{29.21}\right) = (0.030, 0.190, 0.88)
\]

\[
S_6 = (2.76, 5.37, 26, 61) \times \left(\frac{1}{166.02}, \frac{1}{68.37}, \frac{1}{29.21}\right) = (0.020, 0.080, 0.91)
\]

\[
S_7 = (5.16, 10, 54, 34, 69) \times \left(\frac{1}{166.02}, \frac{1}{68.37}, \frac{1}{29.21}\right) = (0.030, 0.15, 1.19)
\]

The \(Si\) vectors in fuzzy analytical hierarchy process algorithm will be compared using the following formula:
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Where:

\[ S_1 = (l_1, m_1, u_1) \]
\[ S_2 = (l_2, m_2, u_2) \]

Therefore, according to the above formula the S1 – S5 vectors will be compared as below:

\[ V(S_1 \geq S_2) = 1 \]
\[ V(S_2 \geq S_1) = \frac{0.04 - 0.72}{(0.13 - 0.72) - (0.27 - 0.04)} = 0.83 \]

\[ V(S_1 \geq S_3) = 1 \]
\[ V(S_2 \geq S_3) = 1 \]
\[ V(S_2 \geq S_4) = \frac{0.03 - 0.72}{(0.13 - 0.72) - (0.27 - 0.03)} = 0.83 \]

\[ V(S_1 \geq S_5) = 1 \]
\[ V(S_2 \geq S_5) = \frac{0.03 - 0.72}{(0.13 - 0.72) - (0.19 - 0.03)} = 0.92 \]

\[ V(S_1 \geq S_6) = 1 \]
\[ V(S_2 \geq S_6) = 1 \]
\[ V(S_2 \geq S_7) = \frac{0.03 - 0.72}{(0.13 - 0.72) - (0.15 - 0.03)} = 0.97 \]

\[ V(S_3 \geq S_1) = \frac{0.04 - 0.59}{(0.10 - 0.59) - (0.27 - 0.04)} = 0.76 \]
\[ V(S_4 \geq S_1) = 1 \]

\[ V(S_3 \geq S_2) = \frac{0.01 - 0.59}{(0.10 - 0.59) - (0.13 - 0.01)} = 0.95 \]
\[ V(S_4 \geq S_2) = 1 \]

\[ V(S_3 \geq S_4) = \frac{0.03 - 0.59}{(0.12 - 0.81) - (0.27 - 0.03)} = 0.60 \]
\[ V(S_4 \geq S_3) = 1 \]

\[ V(S_3 \geq S_5) = \frac{0.03 - 0.59}{(0.10 - 0.59) - (0.08 - 0.03)} = 0.86 \]
\[ V(S_4 \geq S_5) = 1 \]

\[ V(S_3 \geq S_6) = 1 \]
\[ V(S_4 \geq S_6) = 1 \]

\[ V(S_3 \geq S_7) = \frac{0.03 - 0.59}{(0.10 - 0.59) - (0.15 - 0.03)} = 0.92 \]
\[ V(S_4 \geq S_7) = 1 \]

\[ V(S_5 \geq S_1) = \frac{0.04 - 0.88}{(0.19 - 0.88) - (0.27 - 0.04)} = 0.91 \]

\[ V(S_5 \geq S_2) = 1 \]
\[ V(S_5 \geq S_3) = 1 \]

\[ V(S_5 \geq S_4) = \frac{0.03 - 0.88}{(0.19 - 0.88) - (0.27 - 0.03)} = 0.91 \]

\[ V(S_5 \geq S_6) = 1 \]
\[ V(S_5 \geq S_7) = 1 \]
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\[ V(S6 \geq S1) = \frac{0.04 - 0.91}{(0.08 - 0.91) - (0.27 - 0.04)} = 0.82 \]

\[ V(S6 \geq S2) = \frac{0.01 - 0.91}{(0.08 - 0.91) - (0.13 - 0.01)} = 0.95 \]

\[ V(S6 \geq S3) = \frac{0.01 - 0.91}{(0.08 - 0.91) - (0.10 - 0.01)} = 0.98 \]

\[ V(S6 \geq S4) = \frac{0.03 - 0.91}{(0.08 - 0.91) - (0.27 - 0.03)} = 0.82 \]

\[ V(S6 \geq S5) = \frac{0.03 - 0.91}{(0.08 - 0.91) - (0.19 - 0.03)} = 0.89 \]

\[ V(S6 \geq S7) = \frac{0.03 - 0.91}{(0.08 - 0.91) - (0.15 - 0.03)} = 0.93 \]

\[ V(S7 \geq S1) = \frac{0.04 - 1.19}{(0.15 - 1.19) - (0.27 - 0.04)} = 0.91 \]

\[ V(S7 \geq S2) = 1 \]

\[ V(S7 \geq S3) = 1 \]

\[ V(S7 \geq S4) = \frac{0.03 - 1.19}{(0.15 - 1.19) - (0.27 - 0.03)} = 0.91 \]

\[ V(S7 \geq S5) = \frac{0.03 - 1.19}{(0.15 - 1.19) - (0.19 - 0.03)} = 0.97 \]

\[ V(S7 \geq S6) = 1 \]

Then, the d (I) s’ Values are made as below:

\[ d'(I1) = MIN(S1 \geq S2, S3, S4, S5, S6, S7) = MIN(1, 1, 1, 1, 1, 1) = 1 \]

\[ d'(I2) = MIN(S2 \geq S1, S3, S4, S5, S6, S7) = MIN(0.83, 1, 0.83, 0.92, 1, 0.97) = 0.83 \]

\[ d'(I3) = MIN(S3 \geq S1, S2, S4, S5, S6, S7) = MIN(0.76, 0.95, 0.60, 0.86, 1, 0.92) = 0.60 \]

\[ d'(I4) = MIN(S4 \geq S1, S2, S3, S5, S6, S7) = MIN(1, 1, 1, 1, 1, 1) = 1 \]

\[ d'(I5) = MIN(S5 \geq S1, S2, S3, S4, S6, S7) = MIN(0.91, 1, 1, 0.91, 1, 1) = 0.91 \]

\[ d(I6) = MIN(S6 \geq S1, S2, S3, S4, S5, S7) = MIN(0.82, 0.95, 0.98, 0.82, 0.89, 0.93) = 0.82 \]

\[ d(I7) = MIN(S7 \geq S1, S2, S3, S4, S5, S6) = MIN(0.91, 1, 1, 0.91, 0.97, 1) = 0.91 \]

\[ W' = (1, 0.83, 0.60, 1, 0.91, 0.82, 0.91)^T \]

\[ W = (0.16, 0.14, 0.09, 0.16, 0.15, 0.13, 0.15) \]
Thus, based on FAHP method, companies will be prioritized as shown in Table 8.

Table 8: The final matrix of company prioritization according to the International Economics criterion

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Weight criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>0.16</td>
</tr>
<tr>
<td>S2</td>
<td>0.14</td>
</tr>
<tr>
<td>S3</td>
<td>0.09</td>
</tr>
<tr>
<td>S4</td>
<td>0.16</td>
</tr>
<tr>
<td>S5</td>
<td>0.15</td>
</tr>
<tr>
<td>S6</td>
<td>0.13</td>
</tr>
<tr>
<td>S7</td>
<td>0.15</td>
</tr>
</tbody>
</table>

4.2 Determination of Optimal Stock Portfolio:

After prioritizing the companies based on the chosen criteria, the final step was to select the stock portfolio considering all the identified criteria. For this purpose, the matrix of "prioritizing the companies according to the criteria" is multiplied by the matrix of "prioritization of the criteria"; this leads to:

Table 9: The matrix of "prioritizing the companies according to the criteria"

```
<table>
<thead>
<tr>
<th>Profit</th>
<th>international</th>
<th>management</th>
<th>quality</th>
<th>international</th>
<th>efficiency</th>
<th>transparency</th>
<th>competitiveness</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.14</td>
<td>0.18</td>
<td>0.14</td>
<td>0.16</td>
<td>0.18</td>
<td>0.16</td>
<td>0.18</td>
<td>0.33</td>
</tr>
<tr>
<td>0.14</td>
<td>0.15</td>
<td>0.14</td>
<td>0.16</td>
<td>0.12</td>
<td>0.14</td>
<td></td>
<td>0.17</td>
</tr>
<tr>
<td>0.17</td>
<td>0.09</td>
<td>0.15</td>
<td>0.09</td>
<td>0.13</td>
<td>0.15</td>
<td>0.06</td>
<td>0.14</td>
</tr>
<tr>
<td>0.18</td>
<td>0.16</td>
<td>0.17</td>
<td>0.16</td>
<td>0.14</td>
<td>0.16</td>
<td>0.22</td>
<td>0.13</td>
</tr>
<tr>
<td>0.14</td>
<td>0.20</td>
<td>0.16</td>
<td>0.15</td>
<td>0.14</td>
<td>0.14</td>
<td>0.15</td>
<td>0.08</td>
</tr>
<tr>
<td>0.11</td>
<td>0.15</td>
<td>0.12</td>
<td>0.13</td>
<td>0.14</td>
<td>0.14</td>
<td>0.10</td>
<td>0.08</td>
</tr>
<tr>
<td>0.12</td>
<td>0.07</td>
<td>0.12</td>
<td>0.15</td>
<td>0.11</td>
<td>0.13</td>
<td>0.15</td>
<td>0.07</td>
</tr>
</tbody>
</table>
```

In other words, in accordance with the method presented, if an investor would like to invest 100 units in the aforesaid companies, the best option would be a 15.7% investment in Iran Pipe & Machine mfg, 14.2% investment in Behshahr Industrial Co, 13.1% investment in Eghtesad Novin Bank, 17.1% investment in Chadormalu Mining and Industrial Co, 15.9% investment in Mines and Metals Development Investment Co, 12.5% investment in Pars Daru Holding and a 11.8% investment in Farsit Doroud Co. (total of 100.3% due to rounding).

As can be seen, regarding the criteria identified in the previous stage, classification of the selected companies will be different from the initial classification performed by the stock organization, considering the quantitative criterion (total return).
5. Conclusions

The main purpose in this study was to select an optimal portfolio by considering simultaneously the quantitative and qualitative criteria that are effective on the investors’ preferences using a verified and proper scientific method.

According to the results, the selection of an optimal portfolio is dependent on different factors including extrinsic and intrinsic groups, as well as those associated with the investors’ objectives. Among the intrinsic factors and those associated with the investors’ objectives, the most significant one is profitability. Among the extrinsic factors, international risk is the most significant one. The highest weight is related to the intrinsic factors and those associated with the investors’ main objective (profitability) that is available for the companies. Thus, it is better for the companies to pay particular attention to these factors (available factors) and to improve their positions among all other companies.

Moreover, this study also tried to identify all effective variables on the stock portfolio selection as much as possible, to determine the most significant criteria viewed by experts of the stock exchange as effective, and then to select the stock portfolio based on such criteria. Thus according to the criteria selected, the investors can take their own portfolio under relative control. However, because investment is a complicated process and many variables might have an effect on it, the investors should take all requirements into consideration when using such an approach.

In sum, in portfolio selection, qualitative factors as well as quantitative factors will have a direct impact on portfolio selection. In addition, regarding that in stock portfolio selection, humans and their subjective judgments are the main factors in selecting the companies and that the fuzzy options are also human language, where vagueness and linguistic variables are taken into account, using this method is recommended in portfolio selection.

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